TURBULENT FLOW IN A CYLINDRICAL CHANNEL

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The distribution function for turbulent frictional stress is proposed for flow in a cylindrical channel. Formulas are obtained for the calculation of the velocity distribution according to the well-known law of hydraulic drag of the channel.

Stabilized turbulent flow of liquid with a constant viscosity in a cylindrical channel is described by the equation

$$\frac{1}{Rr}\frac{d}{dr}[r\overline{(\rho u)'v'}] = \frac{\mu}{R^2 r}\frac{d}{dr}\left(r\frac{d\bar{v}}{dr}\right) - \frac{1}{l}\frac{dP}{dz} \tag{1}$$

with the boundary condition

$$\bar{v}(1) = 0, \quad \frac{d\bar{v}}{dr}(0) = 0. \tag{2}$$

It is known [1] that the distribution of the turbulent tangential frictional stress $q_t = \overline{(\rho u)'v'}$ must satisfy the conditions

$$q_t(1) = 0, \quad q_t(0) = 0.$$

The distribution q_t will be sought here in the form of the function

$$q_t = \frac{\mu\tau}{2R} r\bar{v}, \tag{3}$$

satisfying the above conditions. Parameter τ is determined from experimental data. Taking account of Eq. (3), Eq. (1) reduces to

$$x\frac{d^2W}{dx^2} + (1-x)\frac{dW}{dx} - W = 0, \tag{4}$$

where

$$W = U\tau - C, \quad C = -\frac{R^2}{\bar{v}_0 l\mu}\frac{dP}{dz}, \quad x = \frac{\tau r^2}{4}.$$

The solution of Eq. (4) is [2]

$$W = \text{const}\, e^x. \tag{5}$$

Taking account of Eqs. (5) and (2), it follows that

$$U = 1 - \frac{1-\exp x}{1-\exp x_0}, \quad x_0 = \frac{\tau}{4}, \tag{6}$$

$$C = -\frac{\tau \exp x_0}{1-\exp x_0}. \tag{7}$$

Using the distribution in Eq. (6), it is found that

$$\frac{dU}{dr} = f(r) = \frac{\tau r \exp x}{2[1-\exp x_0]}, \tag{8}$$

$$\gamma = \int_0^1 Urdr = -\frac{2}{\tau}\left\{\frac{x_0 \exp x_0}{1-\exp x_0} + 1\right\}. \tag{9}$$

Making use of the definition of C_f, it is possible to write

$$C_f = \frac{2f(1)}{\gamma \,\text{Re}}. \tag{10}$$

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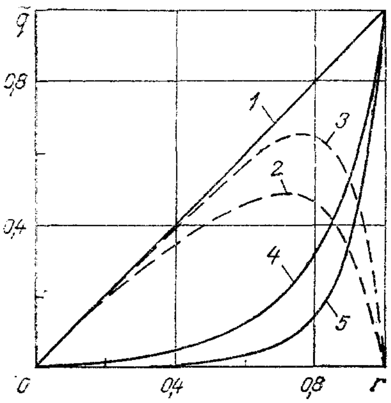

Fig. 1

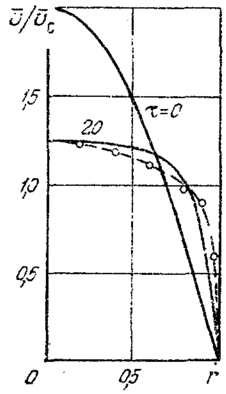

Fig. 2

Fig. 1. Distribution of the relative tangential frictional stress: 1) $\tilde{q}$; 2, 3) $\tilde{q}_t$; 4, 5) $\tilde{q}_M$. Curves 2, 4 and 3, 5 are calculated for τ = 10 and 20, respectively.

Fig. 2. Relative-velocity distribution.

As is known, experimental data on C_f are generalized by the following relation

$$C_f = A\,\mathrm{Re}^{-m}, \tag{11}$$

where A and m are empirical constants.

From Eqs. (10) and (11), an equation is found for determining the value of τ from the specified number Re

$$-\frac{f(1)}{\gamma} = \frac{A}{2}\,Re^{1-m}. \tag{12}$$

Thus, the solution obtained allows the profile of the velocity distribution and the frictional stress to be calculated with a known law of variation of the hydraulic drag of the channel.

It is found from Eqs. (6)-(10) that

$$\lim_{\tau\to 0} U = 1 - r^2, \quad \lim_{\tau\to 0} C = 4, \quad \lim_{\tau\to 0} f(1) = -2, \quad \lim_{\tau\to 0} \gamma = 0.25,$$

$$\lim_{\tau\to 0} C_f = \frac{16}{\mathrm{Re}}.$$

Hence, in the particular case when τ = 0, the given solution gives the well-known results for laminar liquid flow.

The distribution of the relative values of the frictional stress is shown in Fig. 1; these are the turbulent, molecular, and total values, given by the formulas

$$\tilde{q}_t = -\frac{\tau}{2f(1)}\,rU, \quad \tilde{q}_M = \frac{rf(r)}{f(1)}, \quad \tilde{q} = \tilde{q}_t + \tilde{q}_M. \tag{13}$$

As is evident, the total frictional stress in any conditions of flow increases linearly from zero to unity on moving toward the channel wall. This result precisely coincides with the well-known results of [1, 3]. It is also evident from Fig. 1 that, in turbulent flow conditions, the molecular friction is small in the central region of the channel, and the total frictional stress is mainly determined by the turbulent mixing of the liquid. With increase in r, turbulent friction increases, reaching a maximum in the peripheral region of the channel, and then sharply declines in the wall layer to zero at r = 1. The molecular friction rises monotonically on moving toward the channel wall, and the most rapid increase in $\tilde{q}_M$ occurs in the wall layer of the flow, reaching a value of unity at r = 1.

The relative-velocity profiles at τ = 0 and τ = 20 are shown in Fig. 2, where the points correspond to calculation by a logarithmic law and are taken from [1]. It is evident that, with increase in Re, the profile of $\bar{v}(r)$ becomes fuller and, as follows from Eqs. (5)-(8) and (14), $\bar{v}(0) \to \bar{v}_c$ as Re → ∞. It is also evident that the relative-velocity profile calculated from Eq. (6) is in good agreement with the data of [1].

Note that the logarithmic and power-law profiles of the velocity do not satisfy the physical condition $d\bar{v}/dr = 0$ at $r = 0$ [1, 3]. This deficiency is eliminated here.

Thus, the function in Eq. (3) leads to a result in agreement with the well-known laws of turbulent friction.

NOTATION

R, l, radius and length of the channel; r, z, cylindrical coordinates, referred to R and l, respectively; v, gas velocity; v_0, gas velocity at the channel axis; $U = v/v_0$; C_f, friction coefficient.

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TWO-DIMENSIONAL FLOW OF VISCOUS FLUID BETWEEN CYLINDRICAL ROLLERS ROTATING IN OPPOSITE DIRECTIONS

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A method of calculating the hydrodynamic parameters of two-dimensional flow of a viscous fluid through a channel formed by rotating cylinders is described.

An important role in the reprocessing of polymer materials in rolling machines (cylinders, calenders) is played by the flow of the viscous fluid through the gap between oppositely rotating cylindrical rollers. The polymer between rollers is usually in the molten state, characterized by complex hydrodynamic effects which influence the quality of the end product. These effects include a "rotating stock" of material within the deformation zone, which appears when the equipment is heavily loaded. In order to quantitatively estimate the flow characteristics under given technological conditions in a given equipment, it is necessary to mathematically describe the process and to construct an algorithm for calculating the distribution of the velocity components at which particles of the material move through the deformation zone as well as the integral characteristics related to this distribution.

The flow region of the material is shown schematically in Fig. 1.

In order to describe the process theoretically, it is necessary to solve the complete system of two-dimensional Navier–Stokes equations with uniqueness conditions stipulated for the given curvilinear channel.

It is well known that polymer materials are mostly nonlinearly viscous media and, therefore, the Newton hypothesis of friction is inadequate for their description, which limits the practical use of the method which will be described here. We will nevertheless assume first that the material to be reprocessed is Newtonian fluid, the purpose being to simplify the development of this method of calculating a two-dimensional flow through a curvilinear channel and facilitating the test calculations. It will be assumed, furthermore, that the flow is steady and isothermal, also that the rate of material processing is held constant by the feeder band (Fig. 1a) moving around a roller until it enters the gap between rollers as a solid body at the same angular velocity. The flow region is bounded by the rollers on its left-hand and right-hand sides and by the exit coordinate from below, this coordinate being easily calculated from the given entrance coordinate by well-known methods such as those, for instance, which use the condition of constant material flow rate [1].

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